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Dust-acoustic shock waves due to strong correlation among arbitrarily charged dust

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The nonlinear propagation of the dust-acoustic (DA) waves in a strongly coupled dusty plasma containing strongly correlated arbitrarily (positively or negatively) charged dust and weakly correlated Boltzmann electrons and ions has been investigated by employing the generalized hydrodynamic model and the reductive perturbation method. It has been shown that the strong correlation among the charged dust is a source of dissipation and is responsible for the formation of the DA shock waves in such a strongly coupled dusty plasma. It has also been found that the DA shock waves with positive (negative) potential are formed for positively (negatively) charged dust. The basic features of such DA shock waves have been identified. It has been suggested that a laboratory experiment be performed to test the theory presented here.

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More than two decades ago, Ikezi [1] first pointed out that a classical Coulomb plasma with few-micron size negatively charged dust can readily go into a strongly coupled regime. This is due to high charge and low temperature of dust, which make the coupling parameter Γ [$=(q_d^2/a_dT_d)\exp(-a_d/\lambda_D)$, where q_d is the dust grain charge, a_d is the intergrain distance, T_d is the dust temperature in units of the Boltzmann constant, and λ_D is the dusty plasma Debye radius] comparable to one or even much larger than one. This theoretical prediction of Ikezi [1] has been conclusively verified by a series of laboratory experiments [2–4] and simulation studies [5]. These laboratory experimental observations [2–4] and simulation studies [5] clearly demonstrated that a dusty plasma (plasma with few-micron size negatively charged dust) can readily go into a strongly coupled regime and that the charged dust grains organize themselves into crystal structures when $\Gamma > 171$. It has also been observed by further laboratory experiments [6,7] that due to dust grain heating, the dust crystals first melt and then vaporize, leading to the phase transitions (from solid to liquid and then from liquid to gas). These laboratory dusty plasma experiments [2–4,6,7] and simulation studies [5], therefore, provide an excellent opportunity not only to investigate the phase transitions of interest but also to study the wave propagation or wave particle interactions [8–15] in such strongly coupled dusty plasma regimes.

Recently, a strongly coupled dusty plasma containing highly positively charged dust [16], which has been observed in both space [17–20] and laboratory experiments [21–24], has received a considerable interest because of its important role in phase transitions of interest [21–24], in processes occurring in the upper atmosphere [17,25], planetary rings [26], etc. There are three principal mechanisms by which a dust grain in a plasma becomes positively charged [16,20,24]. These are (i) photoemission in the presence of a flux of ultraviolet photons [16,20,27], (ii) thermionic emission induced by radiative heating [20,24,27], and (iii) secondary emission of electrons from the surface of the dust grains [28]. It has been shown by Rosenberg and Mendis [16] that as a result of only the photoemission process a few-micron size dust grain can acquire a positive charge on the order of 10^3-10^5 proton charges. Thus, the Coulomb coupling parameter Γ for few-micron size positively charged dust can easily be order of unity or much larger than unity for which a dusty plasma with positively charged dust also becomes strongly coupled. It is, therefore, important to study the collective processes [particularly the dust-acoustic (DA) waves] associated with strongly coupled positively charged dust and to compare them with those associated with strongly coupled negatively charged dust.

We consider the nonlinear propagation of the DA waves [29] in a strongly coupled dusty plasma whose constituents are arbitrarily (positively or negatively) charged dust, electrons, and ions. Thus, at equilibrium we have $sZ_dn_{d0}+n_{i0}=n_{e0}$, where n_{d0}, n_{i0}, and n_{e0} are the unperturbed dust, ion, and electron number densities, respectively, $s=1$ ($s=-1$) for positively (negatively) charged dust, and Z_d is the number of proton (electron) charges residing on the positive (negative) dust grain surface. We assume that electrons and ions are weakly coupled due to their higher temperatures and smaller electric charges and that dusts are strongly coupled because of their lower temperature and larger electric charge. Thus, in presence of the low phase velocity (in comparison with electron and ion thermal velocities) DA waves, the electron and ion number densities obey the Boltzmann distribution. The dynamics of the nonlinear DA waves in such a strongly coupled dusty plasma is governed by the well-known generalized hydrodynamic (GH) equations [11,30],

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \tag{1}$$

$$D_\tau\left(m_d n_d D_t u_d + q_d n_d \frac{\partial \phi}{\partial x} + \mu_d T_d \frac{\partial n_d}{\partial x}\right) = \eta_l \frac{\partial^2 u_d}{\partial x^2}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e n_{e0}\left[\exp\left(\frac{e\phi}{T_e}\right) - \alpha \exp\left(-\frac{e\phi}{T_i}\right) - \frac{q_d}{e} n_d\right], \tag{3}$$

where n_d is the dust number density, u_d is the dust fluid speed, ϕ is the electrostatic wave potential, $q_d=sZ_de$ is the

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dust charge, t (x) is the time (space) variable, T_e (T_i) is the electron (ion) temperature in units of the Boltzmann constant, m_d is the dust grain mass, e is the magnitude of an electron charge, $\alpha = n_{i0}/n_{e0}$, $D_\tau = 1 + \tau_m \partial/\partial t$, $D_t = \partial/\partial t + u_d \partial/\partial x$, τ_m is the viscoelastic relaxation time, μ_d is the compressibility, and η_l is the longitudinal viscosity coefficient. There are various approaches for calculating these transport coefficients. These have been widely discussed in the literature [11,12,30–32]. The viscoelastic relaxation time τ_m and the compressibility μ_d are given by [30,32]

$$\tau_m = \frac{\eta_l}{n_{d0} T_d}\left[1 - \mu_d + \tfrac{4}{15} u(\Gamma)\right]^{-1}, \quad (4)$$

$$\mu_d = \frac{1}{T_d}\frac{\partial P_d}{\partial n_d} = 1 + \frac{1}{3} u(\Gamma) + \frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial \Gamma}, \quad (5)$$

where $u(\Gamma)$ is a measure of the excess internal energy of the system and is calculated for weakly coupled plasmas ($\Gamma < 1$) as [11,12] $u(\Gamma) \simeq -(\sqrt{3}/2)\Gamma^{3/2}$. To express $u(\Gamma)$ in terms of Γ for a range of $1 < \Gamma < 100$, Slattery *et al.* [31] analytically derived a relation

$$u(\Gamma) \simeq -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81, \quad (6)$$

where a small correction term due to finite number of particles is neglected. The dependence of the other transport coefficient η_l on Γ is somewhat more complex and cannot be expressed in such a closed analytical form. However, tabulated and graphical results of their functional behavior derived from molecular dynamic simulations, and a variety of statistical schemes is available in the literature [30]. To derive a dynamical equation for the DA shock waves from our basic Eqs. (1)–(3), we employ the reductive perturbation technique [33]. We first introduce the stretched coordinates [33,34]

$$\xi = \epsilon(x - V_p t), \qquad \tau = \epsilon^2 t, \quad (7)$$

where ϵ is a smallness parameter measuring the weakness of the dispersion and V_p is the phase speed of the DA waves, and expand the variables n_d, u_d, and ϕ about the unperturbed states in power series of ϵ, viz.,

$$n_d = n_{d0} + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \cdots, \quad (8)$$

$$u_d = \epsilon u_d^{(1)} + \epsilon^2 u_d^{(2)} + \cdots, \quad (9)$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots. \quad (10)$$

Now, using Eqs. (7)–(10) in Eqs. (1)–(3), one can easily develop different sets of equations in various powers of ϵ. To the lowest order in ϵ [i.e., taking the coefficients of ϵ^2 from both sides of Eqs. (1) and (2) and ϵ from both sides of Eq. (3)], one can obtain a set of coupled equations for $n_d^{(1)}$, $u_d^{(1)}$, and $\phi^{(1)}$. The latter can be solved to obtain V_p as

$$\frac{V_p}{C_d} = \sqrt{\frac{s(1-\alpha)}{1+\alpha\beta} + \frac{\mu_d \sigma}{Z_d}}, \quad (11)$$

where $C_d = (Z_d T_e/m_d)^{1/2}$, $\beta = T_e/T_i$, and $\sigma = T_d/T_e$. We note here that $\alpha < 1$ ($\alpha > 1$) for positively (negatively) charged dust (since $n_{e0} = n_{i0} + s Z_d n_{d0}$), $\beta \geq 1$, and $0 \leq \sigma < 1$. Equation (11) represents the linear dispersion relation for the DA waves in which the dust mass provides the inertia and the electron and ion thermal pressures provide the restoring force. It is obvious from Eq. (11) that the phase speed (V_p) is increased by the dust temperature (T_d). However, in most space and laboratory dusty plasma situations, the role of dust temperature is insignificant since $\sigma \ll 1$ and $Z_d \gg 1$.

To the next order in ϵ [i.e., taking the coefficients of ϵ^3 from both sides of Eqs. (1) and (2) and ϵ^2 from both sides of Eq. (3)], one can obtain another set of coupled equations for $n_d^{(2)}$, $u_d^{(2)}$, and $\phi^{(2)}$, which, along with the first set of coupled linear equations for $n_d^{(1)}$, $u_d^{(1)}$, and $\phi^{(1)}$, reduce to a nonlinear dynamical equation of the form

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} = C\frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, \quad (12)$$

where the nonlinear coefficient A and the dissipation coefficient C are given by

$$A = sf\sqrt{\frac{Z_d e^2}{m_d T_e}}, \quad (13)$$

$$C = \frac{1}{2}\frac{\eta_l}{n_{d0} m_d}, \quad (14)$$

$$f = \frac{C_d}{V_p}\left[1 + \alpha\frac{1+\beta(2+\beta)}{2(1+\alpha\beta)^2} + \frac{3}{2}\sigma\frac{\mu_d(1+\alpha\beta)}{sZ_d(1-\alpha)}\right]. \quad (15)$$

Equation (12) is the well-known Burger equation describing the nonlinear propagation of the DA waves in the dusty plasma under consideration. It is obvious from Eqs. (12) and (14) that the dissipative term, i.e., the right-hand side of Eq. (12), is due to the strong correlation among the charged dust. It is clear that $A > 0$ for positively charged dust ($s = 1$ and $\alpha < 1$) and $A < 0$ for negatively charged dust ($s = -1$ and $\alpha > 1$).

A Korteweg–de Vries (K-dV)–Burger equation was derived by Shukla and Mamun [13] in order to study the nonlinear propagation of the DA waves associated with strongly coupled negatively charged dust. They [13] used different stretching coordinates [viz., $\xi = \epsilon^{1/2}(x - V_p t)$, $\tau = \epsilon^{3/2} t$] along with an additional assumption $\eta_l = \epsilon^{1/2}\eta_0$. The nonlinear coefficient (A) of K-dV-Burger equation derived by Shukla and Mamun [13] was found to be negative, and the dissipative coefficient was found to be a function of the stretching parameter (ϵ). Therefore, our present work, where the limitation $\eta_l = \epsilon^{1/2}\eta_0$ is overcome by using a suitable stretched coordinates (which are different from those used by Shukla and Mamun [13]), is different from the work of Shukla and Mamun [13] not only from the physical point of view but also from the view of the mathematical approach.

We are now interested in looking for the stationary shock wave solution of Eq. (12) by introducing $\zeta = \xi - U_0 \tau'$ and $\tau' = \tau$, where U_0 is the shock wave speed (in the reference frame). This leads us to write Eq. (12), under the steady state condition ($\partial/\partial \tau' = 0$), as

$$-U_0\frac{\partial\phi^{(1)}}{\partial\zeta}+A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\zeta}=C\frac{\partial^2\phi^{(1)}}{\partial\zeta^2}. \tag{16}$$

It can be easily shown [35] that Eq. (16) describes the shock waves whose speed U_0 (in the reference frame) is related to the extreme values $\phi^{(1)}(-\infty)$ and $\phi^{(1)}(\infty)$ by $\phi^{(1)}(\infty)-\phi^{(1)}(-\infty)=2U_0/A$. Thus, under the condition that $\phi^{(1)}$ is bounded at $\zeta=\pm\infty$, the shock wave solution of Eq. (16) is [35]

$$\phi^{(1)}=\phi_0^{(1)}[1-\tanh(\zeta/\Delta)], \tag{17}$$

where $\phi_0^{(1)}=U_0/A$ and $\Delta=2C/U_0$ are, respectively, the height and the thickness of the shock waves. It is obvious that the formation of such shock waves is due to the strong correlation among the charged dust and that the shock waves are associated with $\phi^{(1)}>0$ ($\phi^{(1)}>0$) for positively (negatively) charged dust since $A>0$ ($A<0$) for positively (negatively) charged dust. It clear that the shock thickness ($\Delta=\eta_l/U_0\rho_{d0}$) is directly proportional to the longitudinal viscosity coefficient (η_l) and is inversely proportional to the equilibrium dust grain mass density ($\rho_{d0}=n_{d0}m_d$) for both positively and negatively charged dusts.

To interpret the variation in the shock height ($\phi_0^{(1)}$) with different dusty plasma parameters analytically, one can easily neglect σ/Z_d for both space and laboratory dusty plasma situations [36–39]. This assumption ($\sigma/Z_d\ll 1$) allows us to express the shock height ($\phi_0^{(1)}=U_0/A$) as

$$\phi_0^{(1)}=\frac{U_0}{sf_0}\sqrt{\frac{m_dT_e}{Z_de^2}}, \tag{18}$$

where

$$f_0=\sqrt{\frac{1+\alpha\beta}{s(1-\alpha)}}\left[1+\alpha\frac{1+\beta(2+\beta)}{2(1+\alpha\beta)^2}\right]. \tag{19}$$

Equations (18) and (19) clearly indicate that the shock waves with positive (negative) potential are formed for positively (negatively) charged dust and that for fixed values of α and β, the shock height is directly proportional to the square root of the dust mass (m_d) and electron temperature T_e but is inversely proportional to the square root of the dust charge (Z_d). It is clear that $f_0\simeq 1$ for an ideal electron-positive dust [16] plasma ($\alpha\simeq 0$). Therefore, for such an electron-dust plasma situation [16], the shock height is approximately $(m_dU_0^2T_e/Z_de^2)^{1/2}\simeq T_e/e$ (where $U_0\simeq C_d$ is used).

We finally consider the more general situation of a dusty plasma with positively and negatively charged dusts and numerically analyze f [given by Eq. (15)] to show how the shock height ($\phi_0^{(1)}$) varies with different dusty plasma parameters (viz., α and β) in general. The variation in the shock height with α and β is obvious from the variation in $1/f$ with α and β shown in Fig. 1 (in the case of positively charged dust) and Fig. 2 (in the case of negatively charged dust). Figure 1 shows that the height of the positive shock profiles (which are formed for positively charged dust) decreases with the increase in both α and β. Figure 2 shows that the height of the negative shock profiles (which are formed for negatively charged dust) decreases with β but increases with α. We have also found that within typical ranges of different dusty plasma parameters [36–42] ($\alpha=0.1-0.9$ for positively charged dust, $\alpha=2-20$ for negatively charged dust, $\beta=1-10$, $\sigma=10^{-4}-10^{-2}$, and $Z_d=10^3-10^6$) the shock height varies from $\sim 0.05T_e/e$ to $\sim 0.7T_e/e$, i.e., from 0.1 to 1.4 V (for $T_e\simeq 2$ eV [37,42]). It should be noted that $\phi_0^{(1)}>0$ for positively charged dust, whereas $\phi_0^{(1)}<0$ for negatively charged dust.

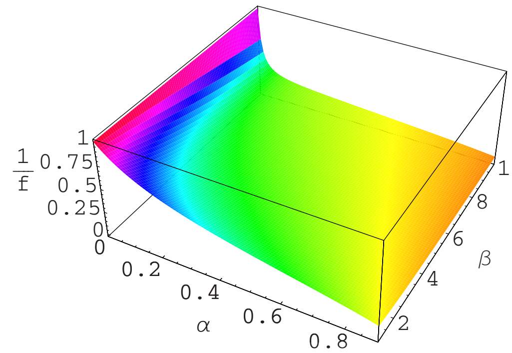

FIG. 1. (Color online) Showing the variation in $1/f$ with α and β for $\sigma=10^{-2}$, $Z_d=10^3$, and $s=1$ (positively charged dust).

Eliasson and Shukla [43] investigated nonstationary DA shocklike structures associated with the self-steepening of the negative potential in a weakly coupled ($\eta_l=0$) dusty plasma containing electrons, ions, and negatively charged dust. This self-steepening of the negative potential, i.e., the formation of shocklike structures (after a certain time) is due to the nonlinear effects. However, to have pure (monotonic or oscillatory) shock waves, there must be a dissipation [35]. The source of dissipation in our case is the strong correlation among the charged dust grains.

To summarize, the nonlinear propagation of the DA waves in a strongly coupled dusty plasma containing Maxwellian electrons and ions and strongly coupled arbitrarily charged dust has been theoretically investigated by employing the GH model and the reductive perturbation method. The results obtained from this theoretical investigation can be pointed out as follows:

(1) The strong correlation among the charged dust is the source of the dissipation and is responsible for the formation of the DA shock waves in such a strongly coupled dusty plasma.

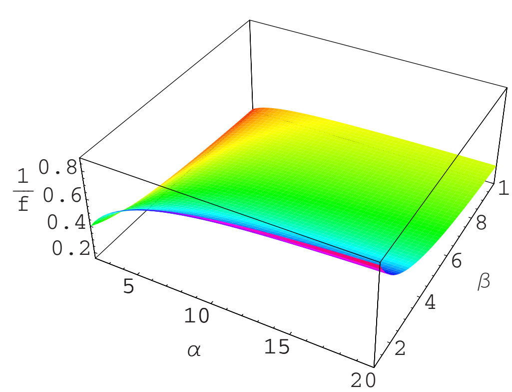

FIG. 2. (Color online) Showing the variation in $1/f$ with α and β for $\sigma=10^{-2}$, $Z_d=10^3$, and $s=-1$ (negatively charged dust).

(2) The DA shock waves with positive (negative) potential are formed for positively (negatively) charged dust.

(3) The shock thickness is directly proportional to the longitudinal viscosity coefficient (η_l) and is inversely proportional to the equilibrium dust mass density (ρ_{d0}).

(4) The shock height is directly proportional to the square root of the dust grain mass (m_d) and electron temperature T_e but is inversely proportional to the square root of the dust grain charge (Z_d).

(5) Within typical ranges of the dusty plasma parameters [36–42] the height of these potential shock profiles varies from 0.1 to 1.4 V for positively charged dust and from −0.1 to −1.4 V for negatively charged dust.

To conclude, we suggest to perform a laboratory experiment to test the theory presented in this work.

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